Multidimensional Cauchy Method and Adaptive Sampling for an Accurate Microwave Circuit Modeling

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***Abstract*—This paper presents an effective generic approach for computer-aided design of microwave circuits. We extend the one-dimensional Cauchy method for frequency-response interpolation to a multidimensional Cauchy interpolation, with respect to both frequency and physical dimensions. This paper also demonstrates the feasibility of applying adaptive sampling to the multidimensional rational-function expansion. Three examples, including optimization and Monte Carlo analysis, have been given to verify the validity of the proposed approach.**

***Index Terms*— Adaptive sampling, CAD, Cauchy method, interpolation, parameter extraction.**

I. Introduction

A NUMBER OF software packages are now commercially available for electromagnetic (EM) simulation of microwave circuits. These packages provide reasonably accurate results allowing designs to be implemented in a cost-effective way. These packages, however, are typically very computation intensive. It has been well recognized that the central processing unit (CPU) time and memory space required to simulate a fully integrated microwave circuits, using these EM simulators, far exceed the capabilities of today's computer workstations.

Over the past years, there has been a strong interest among researchers to circumvent this problem using neural networks [1], space mapping [2], and parameter extraction [3]. The use of the Cauchy method has been also proposed in [4] and [5]. The Cauchy method yields a surprisingly accurate match of the computed points between (interpolated) and even exterior (extrapolated) to the sampled points with the exact solution.

Most of the papers published on application of the Cauchy method, however, deal with one-dimensional (1-D) interpolation, namely frequency-response interpolation. In this paper, we extend the 1-D interpolation for frequency-response interpolation to multidimensional Cauchy interpolation with respect to both frequency and geometrical dimensions. Two different approaches are suggested to achieve a multidimensional approach: a recursive 1-D application of the standard Cauchy method and multidimensional rational-function expansion.

The Cauchy method allows an easy application of adaptive sampling [9]. Adaptive sampling concentrates the computation of samples in regions with highly nonlinear behavior. As a result, samples are taken only in sections where they give additional information, so that this technique reduces the required number of samples even further.

The algorithm for adaptive sampling can be applied to our multidimensional rational-function expansion. Thus, only those samples containing additional information for the interpolation process are taken. When using the recursive Cauchy method, however, adaptive sampling is restricted to one parameter as the samples must fall on a fixed grid.

II. 1-D Cauchy Method

A closer look at system transfer functions, e.g., return and insertion loss, tells us that most of these functions can be represented by a rational polynomial. Consequently, using rational polynomials as interpolation functions yields a much closer representation of the systems response than other schemes, e.g., splines.

The system is described in the form of a fractional polynomial function of numerator order N and denominator order D for one parameter, most often the frequency f by

$$S(f) = \frac{a_0 + a_1 f + a_2 f^2 + \cdots}{1 + b_1 f + b_2 f^2 + \cdots} = \frac{a_0 + \sum_{n=1}^{N} a_n f^n}{1 + \sum_{d=1}^{D} b_d f^d}. \quad (1)$$

In order to determine a function in the form of (1), a number of $k = N + D + 1$ arbitrary sampling points are required. This interpolation scheme is called the Cauchy method or rational-function interpolation.

When using $k = N+D+1$ sample points, a system of linear equations can be set up, whose solution yields the values for the coefficients $a_0, \cdots, a_N$ and $b_1, \cdots, b_D$.

Another faster and more stable algorithm was reported by Stoer and Burlisch [7] of the Neville-type, which performs the interpolation on tabulated data in a recurrence manner. The

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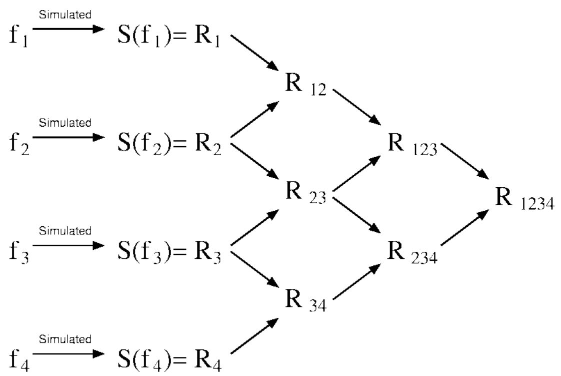

Fig. 1. Tableau for Burlisch–Stoer algorithm.

Burlisch–Stoer algorithm does not require the inversion of a matrix. The algorithm is outlined as follows.

Let R_1 be the value at f_1 of the unique rational function of degree zero (i.e., a constant) passing through the point $(f_1, S(f_1))$. Likewise, define $R_2, R_3, \cdots, R_k$. Now let R_{12} be the rational polynomial of degree one passing through both $(f_1, S(f_1))$ and $(f_2, S(f_2))$. Likewise $R_{23}, R_{23}, \cdots, R_{(k-1)(k)}$. Similarly, for higher order polynomials up to $R_{123\cdots k}$, which is the value of the unique interpolating polynomial through all k points, i.e., the desired answer. The various R's form a tableau with ancestors on the left leading to a single descendant at the extreme right. For example, with $k = 4$, the tableau is as shown in Fig. 1.

The Burlisch–Stoer algorithm is a recurrent way of filling in the numbers in the tableau one column at a time. It is based on the relationship between a child and its parents by [8]

$$R_{i(i+1)\cdots(i+m)} = R_{(i+1)\cdots(i+m)} + \frac{R_{(i+1)\cdots(i+m)} - R_{i\cdots(i+m-1)}}{\frac{f-f_i}{f-f_{i+m}}\left(1 - \frac{R_{(i+1)\cdots(i+m)} - R_{i\cdots(i+m-1)}}{R_{(i+1)\cdots(i+m)} - R_{(i+1)\cdots(i+m-1)}}\right) - 1} \tag{2}$$

It produces the so-called diagonal rational function, with the degree of the numerator and denominator equal (if k is odd) or with the degree of the denominator larger by one (if k is even). For the derivation of the algorithm, refer to Stoer and Burlisch [7].

To demonstrate the efficiency of the 1-D Cauchy method, the results of different interpolation schemes (linear, spline, and Cauchy) are shown in Fig. 2. The response of a four-pole narrow-band planar filter has been sampled at 20 frequency points only. The linear interpolation does not yield much information about the behavior of the circuits response. The spline interpolation provides better correlation due to the smooth curve fitting; however, the curve does not match with the exact solution. The Cauchy method, on the other hand, shows the exact response, even though sampled at a few points only.

Dhaene *et al.* [9] showed that an adaptive sampling scheme can be applied in order to reduce the number of sampling points to the minimum. First, a few samples are taken. Using these samples, two different rational polynomial approximations are computed. These two models are then scanned for the frequency with the biggest mismatch. At this point, the next sample is taken and the procedure is repeated until both models agree.

III. Multidimensional Cauchy Method

The rational-function interpolation can be extended to the interpolation of multidimensional functions. Two new approaches are shown here: a multidimensional recursive Cauchy method and a multidimensional rational-function expansion.

A. Recursive Cauchy Method

The recursive method solves the multidimensional interpolation using a recursive algorithm. The algorithm itself performs a 1-D Cauchy interpolation as described in Section II. From a given set $\mathcal{H}$ of γ sample points and an arbitrary point p^*, the algorithm $\mathbf{C}$ calculates the interpolated function value $S^*(p^*)$.

The set $\mathcal{H}$ is put together by the pairs of sampling points p' to p^γ and their function values S' to S^γ. Thus, $\mathcal{H}$ can be written as

$$\mathcal{H} = \{(p', S'), (p'', S''), (p''', S'''), \cdots, (p^\gamma, S^\gamma)\}. \tag{3}$$

The algorithm C can be defined as a function $\mathbf{C}(\mathcal{H}, p^*)$, which yields the interpolated response S^* for p^* using the samples $\mathcal{H}$

$$\mathbf{C}(p^*, \mathcal{H}) \doteq [(p^*, \{(p', S'), (p'', S''), (p''', S'''), \cdots, (p^n, S^n)\}) \longrightarrow S^*]. \tag{4}$$

Using these definitions, the algorithm can now be extended to multidimensional interpolation. For this purpose, the set of sample points must be extended from the 1-D sample point set $\mathcal{H}$ to a multidimensional sample-point array.

1) Choice of Sample Points in Parameter Space: The sample points in an n-dimensional parameter space are now represented by vectors $\vec{p} = (p_1, p_2, \cdots, p_n)$. For the recursive algorithm, the set of sample points $(\vec{p}, S)$ must fall on a completely filled grid of points. The grid does not have to be equidistant. An example of sample locations in the parameter space are shown in Fig. 3 for two and three parameters.

Each dimension has its own subset of parameter values, as seen from Fig. 3, which do not change for different values of the other parameter dimensions. In the example given, the set of samples in the p_1-dimension has just the four parameter values p'_1 to p''''_1.

2) Algorithm Implementation: The goal is to interpolate the function value S^* of an arbitrary located point $\vec{p}^* = (p_1^*, p_2^*, \cdots, p_n^*)$. The algorithm can be divided into the following three steps.

Step 1) The root process starts with interpolating the point p^* with constant $p_2 = p_2^*$, $p_3 = p_3^*$, etc., parallel to the p_1-axes, shown as a dashed line in Fig. 4(a). This is a 1-D interpolation, so the algorithm $\mathbf{C}$ defined in (4) can be used. This step yields the desired interpolated point

$$S^* = \mathbf{C}([p_1^*, p_2^*, \cdots], \mathcal{A}) \tag{5}$$

where $\mathcal{A}$ is the set of sampling points for p'_1 to p''''_1

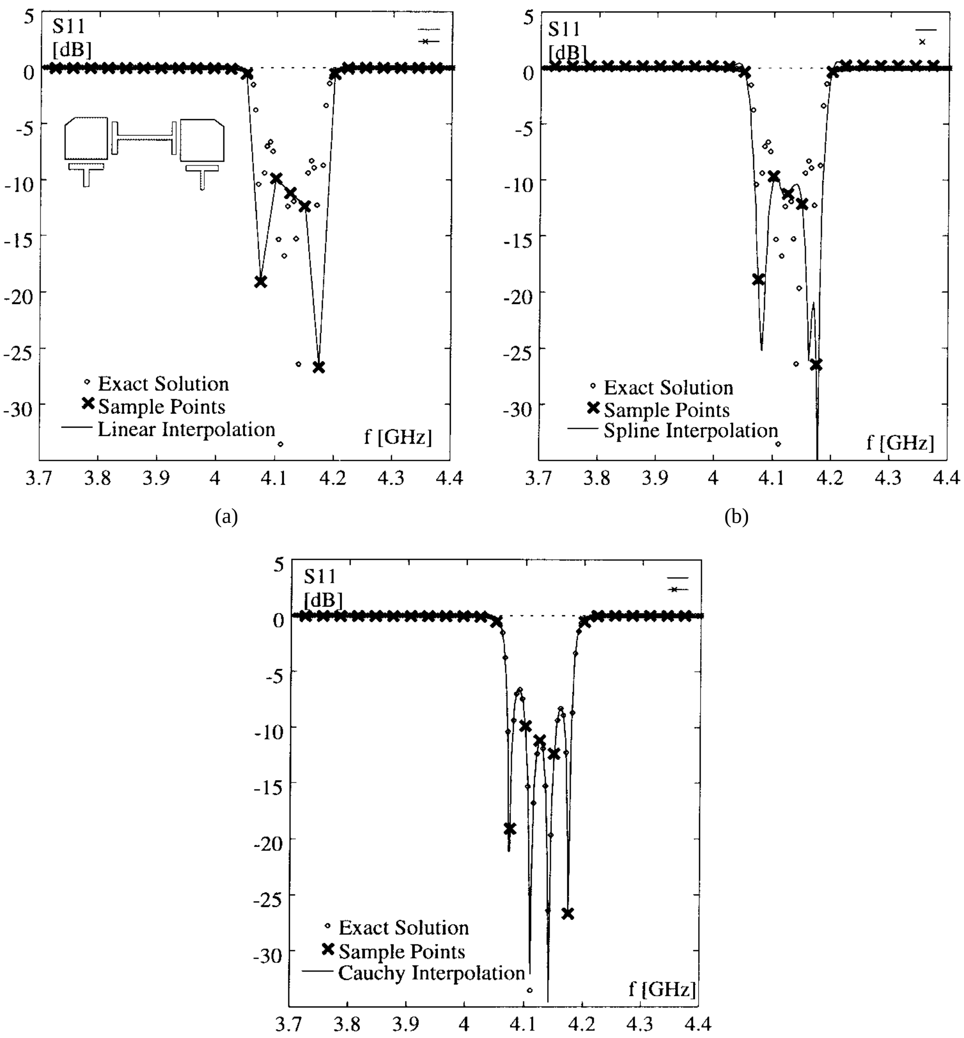

Fig. 2. Interpolation of four-pole filter response. (a) Linear. (b) Cubic spline. (c) Cauchy.

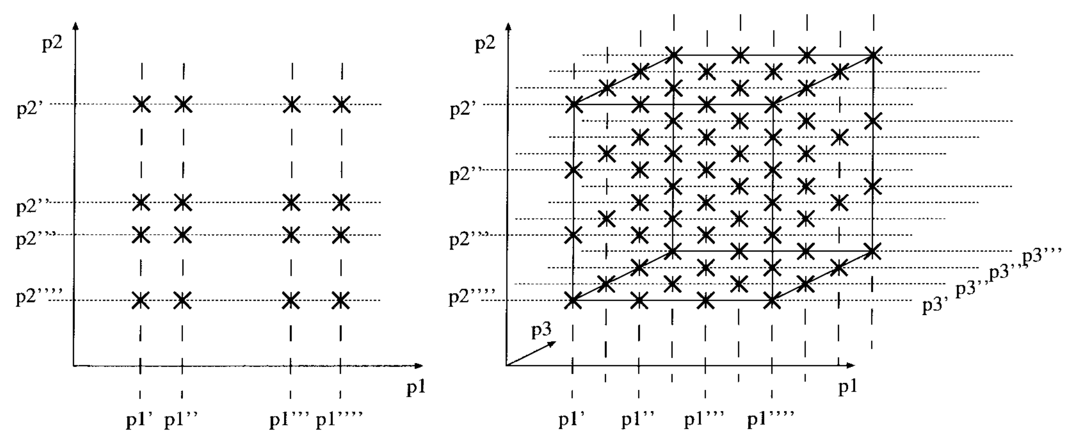

Fig. 3. Sample locations for two- and three-dimensional parameter space.

and $p_2 = p_3, \cdots = \text{const}$. These are the points marked with • in Fig. 4. They may not fall on the grid of known sample points. If that is the case, the algorithm proceeds to Step 2 in order to determine the points •. Otherwise (the points are known), the algorithm proceeds with Step 3.

Step 2) The algorithm calls itself for each of the unknown points •. In the example, the algorithm starts four new child processes

$$\mathbf{C}((p_1', p_2^*), \mathcal{B}_1) \quad \mathbf{C}((p_1'', p_2^*), \mathcal{B}_2)$$
$$\mathbf{C}((p_1''', p_2^*), \mathcal{B}_3) \quad \mathbf{C}((p_1'''', p_2^*), \mathcal{B}_4) \qquad (6)$$

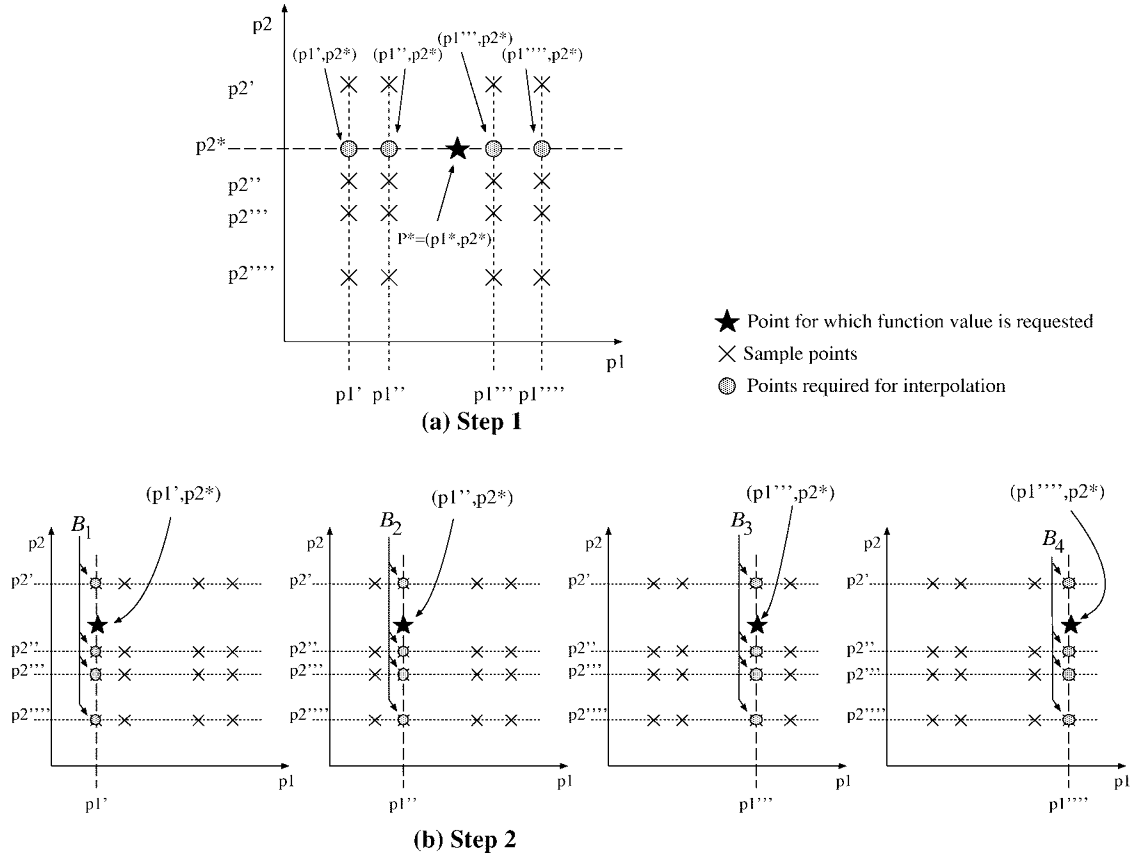

Fig. 4. Steps in recursive Cauchy method.

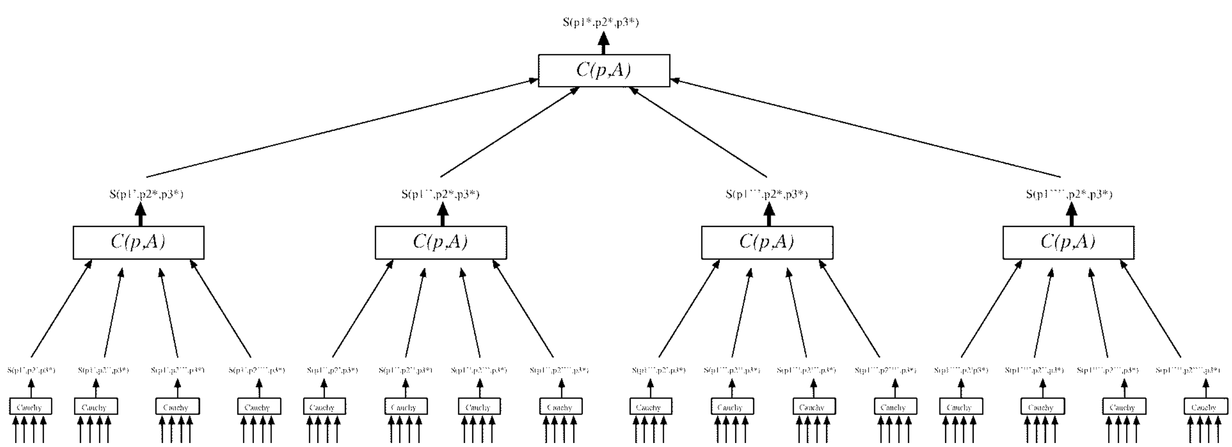

Fig. 5. Inverted tree representation for recursive Cauchy method.

as shown in Fig. 4(b). The $\mathcal{B}$'s are sets of sample points with a fixed value for p_1, as seen from Fig. 4. The interpolation is now performed along the p_2-axis. The routine called is exactly the routine already used in Step 1. The algorithm is thus *recursive*. Again, each subprocess checks if the set $\mathcal{B}$ is from known samples. If not, the algorithm starts another instance of subprocesses in order to interpolate the points included in $\mathcal{B}$ using the next higher dimension. In the example, this would be p_3.

Step 3) In case the subprocess determines that all sample points are known, it calculates the interpolated point using $\mathbf{C}$ and hands it back to the parent process, which requested that point.

Finally, the answer for the root process (5) will be found.

The iterations of the algorithm can be represented in an inverted tree diagram (see Fig. 5). Each process $\mathbf{C}(\mathcal{H}, p^*)$ requests the unknown sample points in its dimension from a number of child processes and hands the result of its interpolation to its parent. The tree is terminated ("leaves") with processes that interpolate from known sample points.

Adaptive sampling can only be applied in the last dimension (the leaves), as all other samples must fall on the grid. This is not a major limitation because most parameters show a slow variation and the most nonlinear parameter can be chosen last.

As mentioned before, the 1-D rational function of (1) can also be expanded to a multidimensional rational function, thus extending the adaptive sampling to more dimensions. An outline of the multidimensional rational-function interpolation is given in Section III-B.

B. Multidimensional Rational-Function Expansion

The multidimensional method can also be implemented by a single multidimensional rational polynomial in the form

$$S(p_1, p_2, p_3, \cdots) = \frac{P_{\text{num}}(p_1, p_2, p_3, \cdots)}{P_{\text{den}}(p_1, p_2, p_3, \cdots)} \tag{7}$$

where $P_{\text{num}}(p_1, p_2, p_3, \cdots)$ and $P_{\text{den}}(p_1, p_2, p_3, \cdots)$ are arbitrary polynomials of the parameters in the numerator and denominator p_1 to p_n, respectively. Using this approach, the coefficients of (7) are determined directly. Sakata [10] showed the extension into two dimension, and the general scheme is discussed shortly here. Equation (7) can be written as

$$S(p_1, p_2, p_3, \cdots) = \frac{a_0 + \sum_{j=1}^{N} a_j P_j(p_1, p_2, \cdots, p_n)}{1 + \sum_{j=1}^{D} b_j P_j(p_1, p_2, \cdots, p_n)} \tag{8}$$

where $P_j(p_1, p_2, \cdots, p_n)$ are the mixed terms of the existing parameters. The general form of the mixed term elements is

$$P_j = \prod_{i=0}^{j} p_i^{u(j)} \tag{9}$$

where $u(j)$ is an integer function of j.

Much attention has to be paid to the choice of the functions $u(j)$. In this paper, the way used is to get all mixed terms of all parameters $p_1, \cdots, p_n$ up to a specified maximum power and then sort by power sums $u_{\text{sum}} = \sum_{i=1}^{n} u_i$. Starting with the lowest power sum, the polynomial is built. Using this scheme, the start of the polynomial for two dimensions is

$$P(p_1, p_2) = a_0 + a_1 p_1 + a_2 p_2 + a_3 p_1 p_2 + a_4 p_1^2 + a_5 p_2^2 + a_6 p_1^2 p_2 + \cdots. \tag{10}$$

This approach yields a linear equation system. Solving this system directly determines the coefficients of (8) and, hence, a *closed-form* and *differentiable* equation of the system's response $S(p_1, p_2, p_3, \cdots)$.

However, it should be mentioned that for large dimensional problems, it would be more efficient to split the problem into several problems of lower order, which are then solved recursively, as shown in Section III-A.

C. Adaptive Sampling

The multidimensional rational-function expansion may utilize adaptive sampling. To do so, the function is approximated with to different integer functions $u_1(j)$ and $u_2(j)$. Using the functions in (9) results in two different approximations: $S_1(\vec{p})$ and $S_2(\vec{p})$. The next sample is taken at the point of biggest mismatch of the approximations $S_1(\vec{p})$ and $S_2(\vec{p})$.

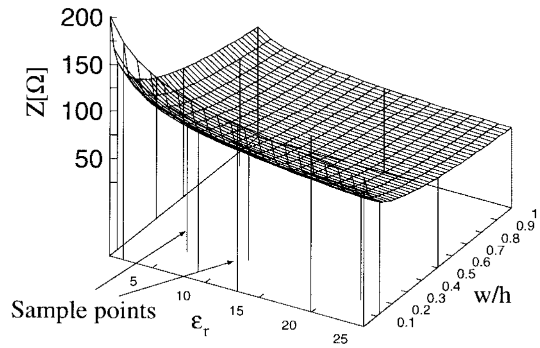

Fig. 6. Impedance of microstrip line as a function of ϵ_r and w/h using multidimensional rational-function expansion.

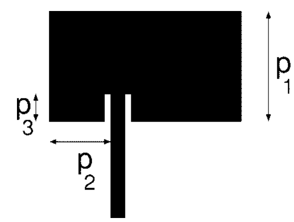

Fig. 7. Microstrip antenna with recessed line feed.

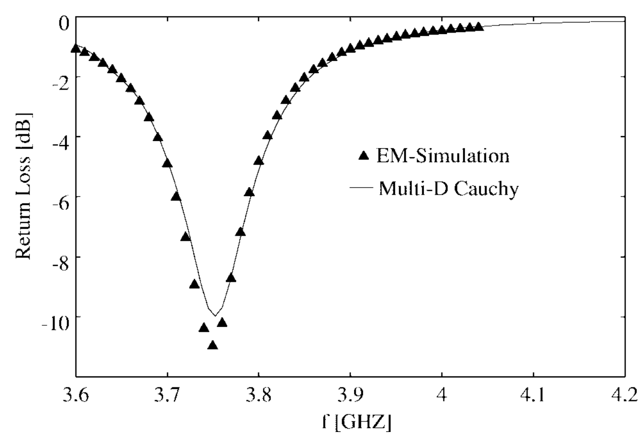

Fig. 8. Return loss of antenna geometry $p_1 = 24.5$ mm, $p_2 = 1.4$ mm, $p_3 = 4.2$ mm, computed by a multidimensional Cauchy method model and full EM simulation.

IV. Examples

Three examples are given here to demonstrate the interpolation using the multidimensional Cauchy method. All EM simulations are performed using the Sonnet *em* planar EM solver. All central processing unit (CPU) times given refer to computations on a Hewlett-Packard K-class machine.

A. Microstrip Line Impedance

The multidimensional rational-function expansion is demonstrated by modeling the line impedance of a microstrip line, with respect to the line's width-to-height ratio w/h and the relative dielectric constant ϵ_r of the substrate.

The modeling algorithm described in Section III-B returns a closed-form rational function of $Z(\epsilon_r, w/h)$.

The samples are determined by the adaptive sampling technique described above. Fig. 6 shows the model approximation

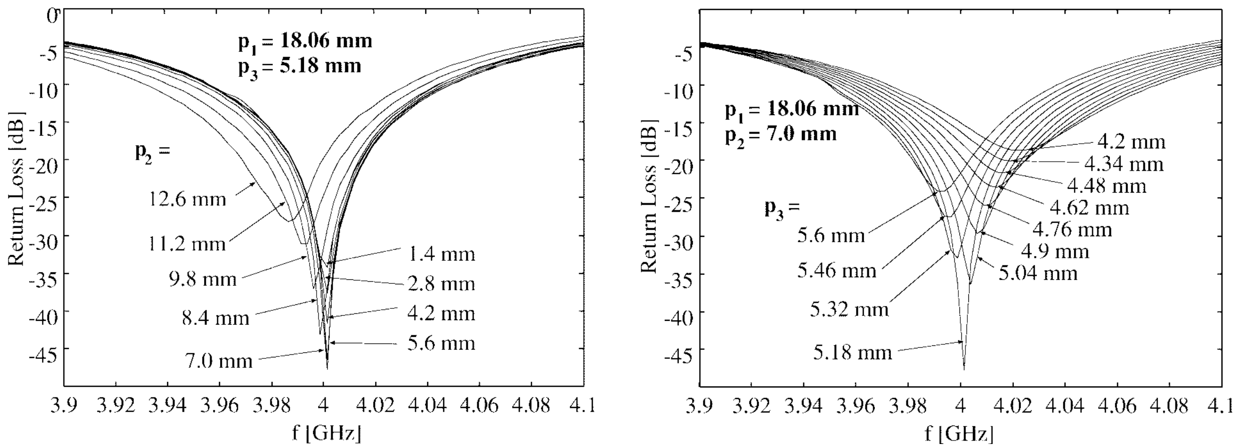

Fig. 9. Effect of inset location on antenna's return loss.

and the 19 adaptively taken samples. The model provides an accuracy within 0.1% error.

B. Recessed-Line-Fed Microstrip Antenna

Heuristic models of microwave circuits are restricted to very basic topologies such as lines, gaps, step, etc. The multidimensional Cauchy method allows the creation of fast and accurate models for any kind of topology as long as a parameterized EM simulation is feasible. Here, the building of a generic model for an recessed-line-fed microstrip antenna in C-band on a 10 mil-Rogers RT/duroid 5870 substrate (see Fig. 7) is discussed. An accurate model for such an element does not exist and, up to now, the design is based on simplified cavity models [12] or an expensive search by a vast number EM simulations, which must be repeated whenever a new slightly modified design is requested.

The multidimensional Cauchy method overcomes these problems. All expensive EM simulations for the microstrip antenna are computed all at once. The problem has four dimensions: the parameters p_1 to p_3 and the frequency f. Five parameter values per dimension are required. Hence, the responses of $5^4 = 625$ samples have to be computed for a complete filled grid of sample points. This data-acquisition phase requires approximately 10 h of CPU time. Even though this initial effort is quite high, it pays off as the established model can be used for a large variety of antenna layouts without any additional expensive computations.

The simulated geometries fall on the following grid:

$$\begin{aligned} p_1 &= 17.5 \text{ mm}, 20.3 \text{ mm}, 23.1 \text{ mm}, 25.9 \text{ mm}, 28.7 \text{ mm} \\ p_2 &= 0.0 \text{ mm}, 2.8 \text{ mm}, 5.6 \text{ mm}, 8.4 \text{ mm}, 11.2 \text{ mm} \\ p_3 &= 2.8 \text{ mm}, 5.6 \text{ mm}, 8.4 \text{ mm}, 11.2 \text{ mm}, 14 \text{ mm} \\ f &= 3.4 \text{ GHz}, 3.7 \text{ GHz}, 4.0 \text{ GHz}, 4.3 \text{ GHz}, 4.6 \text{ GHz}. \end{aligned}$$

The frequency response for the model is verified at an unsampled arbitrary geometry with $p_1 = 24.5$ mm, $p_2 = 1.4$ mm, and $p_3 = 4.2$ mm. The response computed by full EM simulation and by the model is shown in Fig. 8. As seen, the model is in very good agreement with the EM simulated result.

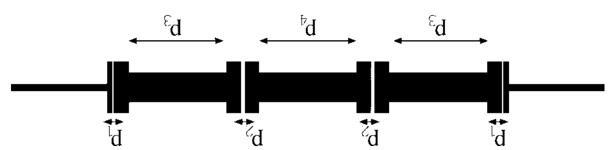

Fig. 10. Layout of three-pole filter showing parameters p_1—p_4.

1) Parameter Plot of Feed Location: Using the multidimensional Cauchy method model, the antenna has been optimized for minimal return loss at 4 GHz. The effects of the feed location on the return loss is shown by a parameter plot of the return loss.

The plots in Fig. 9 contain the information of 2000 frequency points. Obtaining the same information using EM simulators, one has to perform a simulation requiring 33 CPU h. The discussed model computes the same information in less than 4 CPU s. It can be clearly seen that the established model enables the designer to investigate the circuits performance with respect to dimension changes without any additional expensive calculation.

C. Narrow-Band Three-Pole Filter

In this example, the S-parameters of the response of a planar superconductive microstrip filter, as shown in Fig. 10, are interpolated. The five parameters are the four geometrical parameters, namely, the gap and resonator lengths and the frequency. A sample grid with five sample points per dimension for the geometrical parameter is used.

Due to the underlying EM-simulation software, the parameter values are forced to lie on a 1.75-mil grid. The frequency dependency is determined by adaptive sampling, as the last level of the recursive algorithm. The frequency is chosen because, by far, it shows the largest variation of the S-parameter.

Fig. 11 shows the interpolated response in comparison with the exact solution, obtained by a finer meshing and finer frequency stepping. A very good agreement is observed between the interpolated response and exact solution.

The response where the multidimensional Cauchy method is not applied is shown in Fig. 12. Due to the restrictions that all geometrical values have to fall on the 1.75-mil grid, the pa-

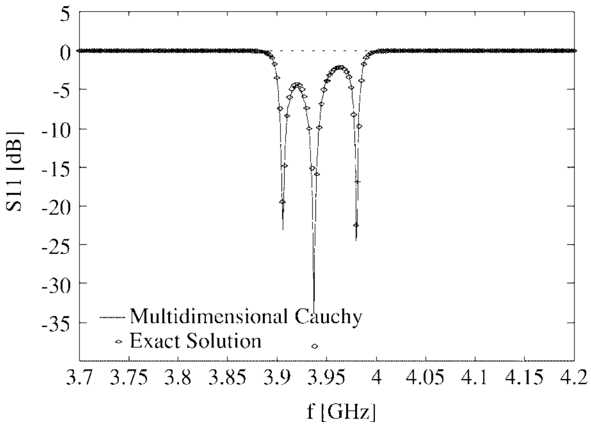

Fig. 11. Interpolated S_{11} parameter using the multidimensional Cauchy method for the parameter values $p_1 = 6.125$, $p_2 = 16.625$, $p_3 = 238.875$, and $p_4 = 238.875$.

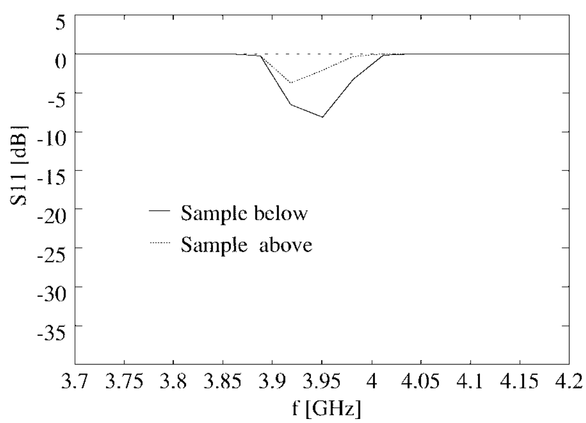

Fig. 12. S_{11} parameter without using the multidimensional Cauchy method, as in Fig. 10, showing the sampled points above and below.

rameter values must be snapped to the next grid point. Fig. 12 shows the response for both rounded-up and rounded-down values for the parameters p_1–p_4. In addition, the frequency resolution is lost, as only the sampled frequencies can be shown. As a result, the two-ripple filter response degrades to a meaningless polygon.

1) Optimization, Monte Carlo Analysis: Using the model, the filter has been optimized with respect to the following specifications:

$$S_{11} < -20 \text{ dB for } 3.9 < f < 3.95$$
$$S_{21} < -20 \text{ dB for } f = 3.85 \text{ GHz and } f = 4.00 \text{ GHz}.$$

The optimization by gradient search requires 19 iterations, which corresponds to 1140 function calls. A full EM simulation would require at least a CPU time of several days, as the circuit's response has to be repeatedly computed on a very fine grid. The multidimensional Cauchy Method solves the problem within 3 CPU min. The response of the optimized filter is shown in Fig. 13.

Having the model on hand, the designer can check the sensitivity of the model by a Monte Carlo analysis. Again, the analysis—known to be expensive or not feasible when EM simulation is used—can be applied with little effort using the predefined model. In Fig. 14, a random variation of ±0.1 mil is added to all geometrical parameters.

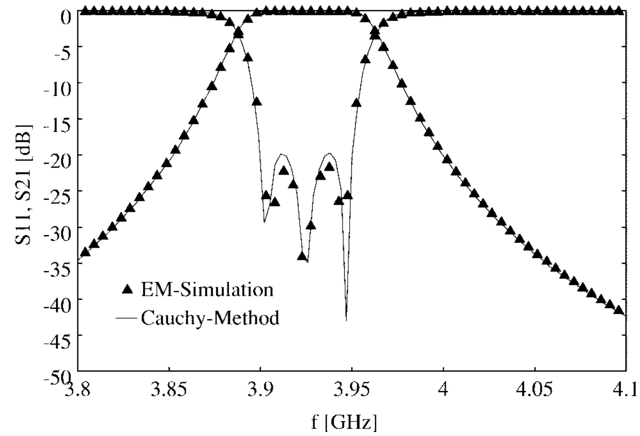

Fig. 13. Response of optimized three-pole filter.

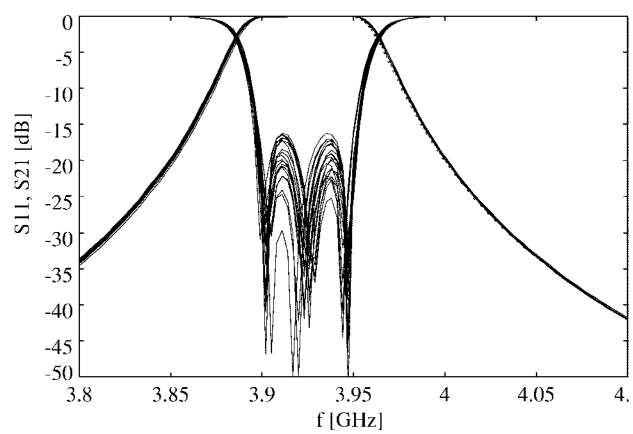

Fig. 14. Monte Carlo analysis of optimized three-pole filter.

Neither optimization nor Monte Carlo analysis can be performed involving direct EM simulation. The number of required points exceeds even the capabilities of current multiprocessor computers. Moreover, the majority of planar EM solvers are not capable of handling geometries with small variations, as all vertices must fall on a relatively rough grid. The multidimensional Cauchy method model overcomes these problems. After one expensive generation of an on-grid database, all following computations of arbitrary circuits variations can be obtained by the inexpensive recursive algorithm.

V. Conclusion

In the past, many publications showed the remarkable reduction of computational cost when the Cauchy method and adaptive sampling is applied to the frequency-response interpolation.

This paper has shown that the method can be extended to the application on multidimensional problems. This can either be done by recursive application of the Cauchy method or by an all-in-one multidimensional rational polynomial approach.

In doing so, similar savings of computational expenses for multidimensional problems can be achieved, as for the 1-D case. In this paper's examples, the authors have shown that a complete and accurate numerical model of a three-pole filter with four geometrical parameters plus frequency dependency can be obtained.

It has also been demonstrated that the developed models can be implemented in an optimization loop for a fast optimization of microwave circuits. Moreover, the authors have shown that the model can be utilized for an inexpensive generation of parameter plots and Monte Carlo analysis with the accuracy of a full EM simulation. An example has also been given to demonstrate the concept of the multidimensional rational-function expansion approach. Both approaches can be combined to tackle large-dimensional problems using adaptive sampling for several parameters at the same time.

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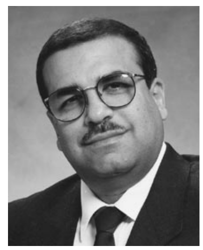

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